## Math 121

1. (10 pts) The stem-and-leaf plot below shows the number of years in the U.S. Senate for each of the current senators. The diagram is a bit too long and narrow. Improve upon it by redrawing it as a histogram using *more than* 5 classes.

Stem	Leaf
0	0000000000122222222444444455666666678888888
1	000000000011223444445556888
2	00000022224666889
3	002449
4	458

*Note*: 1|2 represents 12 years.

- 2. (18 pts) Using the data and graphs of the previous problem, do the following.
  - (a) (6 pts) Find a five-number summary of the data.
  - (b) (6 pts) Draw a modified boxplot of the data.
  - (c) (3 pts) Describe the shape of the distribution, using statistical terminology.
  - (d) (3 pts) If Senator Robert Byrd (48 years in the Senate) were to suddenly retire and be replaced by a new senator, would the resulting distribution have a greater or a smaller standard deviation? Explain.
- 3. (16 pts) A sample of 12 used-car ads in the newpaper had the following asking prices:

6400, 5000, 8000, 5000, 32500, 26900, 36400, 22900, 1500, 5750, 28900, 19000

- (a) (4 pts) Find the mean of the sample.
- (b) (4 pts) Find the standard deviation of the sample.
- (c) (4 pts) Find the standard score (z-score) of 8000.
- (d) (4 pts) Suppose we consider an outlier to be any value whose z-score is greater than 2.5 or less than -2.5. Does this set have any outliers? Justify your answer.
- 4. (8 pts) For the standard normal random variable Z, find the following.
  - (a) (4 pts) P(-2.45 < Z < 1.32).
  - (b) (4 pts) The 45th percentile.

- 5. (16 pts) Suppose that the population of all IQ scores has a normal distribution with a mean of 100 and a standard deviation of 15.
  - (a) (4 pts) What proportion of the population has an IQ score between 90 and 120?
  - (b) (4 pts) What proportion of the population has an IQ score greater than 120?
  - (c) (4 pts) Find the first quartile of IQ scores.
  - (d) (4 pts) To attract the best and the brightest, a certain college decides to give a full scholarship to anyone whose IQ score is in the top 5% of the population. What is the lowest IQ score that will win a full scholarship from this college?
- 6. (8 pts) Let X be a random variable whose probability distribution function is shown in the graph below.
  - (a) (2 pts) What is the height of the graph (at the peak)?
  - (b) (6 pts) Find the probability that X is between 2 and 4.



7. (12 pts) The distribution of a random variable X is either N(20, 5) or N(30, 10). The graphs of the two distributions are shown below. A researcher must decide which is the correct distribution. Let the two hypotheses be

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H_0: X is N(20, 5).
H_1: X is N(30, 10).
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Sampling from this population is extremely expensive, so the researcher will base his decision on a single observation. By selecting one value of X, he will decide which distribution he believes is correct. If the selected value is less than 25, he will accept  $H_0$ . Otherwise, he will reject  $H_0$ .

- (a) (4 pts) Find the value of  $\alpha$ .
- (b) (4 pts) Find the value of  $\beta$ .
- (c) (4 pts) Find the *p*-value of 30.
- 8. (12 pts) Suppose that the proportion of males in a population is p = 0.60.
  - (a) (9 pts) Find the sampling distribution of  $\hat{p}$  for samples of size n = 2. Use a table to describe the distribution.
  - (b) (3 pts) If one random sample of size 2 is taken, what is the probability that  $\hat{p} = 1/2$ ?